DYNAMICAL CYCLES IN CHARGE AND ENERGY FOR IRON IONS ACCELERATED IN A HOT PLASMA

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ABSTRACT

We consider a unified model of Fe ion acceleration in the solar corona. The model comprises charge-changing processes, Coulomb energy losses, and both regular and stochastic acceleration. At a given acceleration scenario, the type of acceleration is found to have a minor effect on the mean charge states, but the shapes of the charge-state distributions produced by regular acceleration and by stochastic acceleration are different. During a continual acceleration at coronal temperatures, iron ions typically follow rising trajectories on the charge-energy plane. These trajectories are situated below the mean equilibrium charge curve defined from the balance of ionization and recombination at fixed energy. During stopping, the iron ions cross the equilibrium charge curve and run through a series of charge states above the mean equilibrium charge at current energy, because the Coulomb deceleration rate significantly exceeds the rate of the ion recombination in a hot plasma. As a result, the variety of possible trajectories on the ion charge-energy plane turns out to be much wider than would be expected based on the equilibrium charge-state approximation. In particular, we find dynamical cycles in charge and energy, so that accelerated and highly stripped ions may reappear at low energies. We also find that the equilibrium charge curve cannot be reproduced without strong reduction in the total number of accelerated particles. This implies that the observed iron charge-state distributions essentially depend on the scenario of their acceleration and transport.

Subject headings: acceleration of particles — Sun: flares — Sun: particle emission

1. INTRODUCTION

Ionization-state measurements are expected to play a key role in deciphering the local conditions at the origin of solar energetic particles (SEPs) as well as the processes involved in their selection, acceleration, and transport (e.g., Möbius et al. 1999). A history of those studies was briefly summarized by Reames, Ng, & Tylka (1999). We would like to focus on the development of theoretical calculations of the SEP charge states. The importance of charge-changing processes for Coulomb energy losses and, by this expedient, for the injection of different ion species into the acceleration regime was recognized in the early days of cosmic-ray research (Ginzburg & Syrovatskii 1964, § 7). Based on experimental data for neutral media, it was accepted that energetic ions rapidly approach an equilibrium value of their mean charge states after traversing a small amount of matter, so that a certain mean charge could be ascribed to a given ion velocity irrespective of the history of ion acceleration/deceleration processes. Such an equilibrium charge was also used for calculations of ion stopping in target matter (effective charge for stopping). In application to solar energetic particles, these issues were discussed by Korchak (1980).

A key contribution to the kinetic modeling of SEP charge states was done by Luhn & Hovestadt (1987), who evaluated energy-dependent rates of energetic ion recombination, through averaging the corresponding cross sections over thermal electron distributions in the ion rest frame. Those rates were employed to calculate mean equilibrium charge states of energetic ions from C through Si. However, the proton-impact ionization was missed. A similar calculation but with proton-impact ionization included was done for Fe ions by Kocharov et al. (2000). A pioneering model of ion acceleration with allowance made for charge-changing processes was proposed by Kurganov & Ostryakov (1991). In that oversimplified model, all energy and charge dependencies in the rates of ionization and recombination were neglected, and by this expedient, an elegant analytic solution was found. More realistic modeling requires numerical treatment (Ostryakov & Stovpyuk 1999; Barghouty & Mewaldt 2000; Ostryakov et al. 2000b). However, the latest papers were largely aimed at fitting particular experimental points but not at comprehensive investigation of the model. For this reason, important properties of the system have been left obscured. There is also an essential distinction between the kinetic modeling method and the equilibrium charge-state approach adopted by Reames et al. (1999), whereas applicability of the different treatments has not been completely understood yet.

In the present paper we consider a unified model of Fe ion acceleration in the solar corona. The model incorporates charge-changing processes, Coulomb energy losses, and both regular and stochastic acceleration. In the aforementioned kinetic models, similar processes were also included, but the results of stochastic acceleration were not compared to the results of regular acceleration, and also, ion phase trajectories were not investigated. That may be justified for a study of a particular SEP event. However, goals of theoretical modeling should not be reduced to the fitting of presently available experimental data but should also include an exploration of general regularities that come from the physics. A proper theoretical analysis of the energy-dependent SEP charge-state models has not been performed yet. The present study is aimed at a comprehensive coverage of the entire parameter region allowed in the solar corona. In particular, we consider the slow acceleration case, where significant interplay between

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2. THE MODEL

We generalize the models previously proposed for ion acceleration in the solar corona by including both regular and stochastic accelerations. We consider the energy range below 100 MeV nucleon\(^{-1}\), where relativistic effects are not essential (e.g., Miller, Guessoum, & Ramaty 1990).\(^3\) In the isotropic case of stochastic acceleration, the distribution function of accelerated particles, \(F(p, t)\), will comply with the diffusion equation in the momentum space (e.g., Tsytovich 1977, § 5.1).

\[
\frac{\partial F}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D_p \frac{\partial F}{\partial p}.
\]

(1)

In the case of nonrelativistic particles, the equation will not change if we substitute momentum \(p\) with the square root of twice the energy per nucleon, \(p \rightarrow V = (2E)^{1/2}\). Then one can reformulate equation (1) in terms of the ion energy per nucleon, \(E\), and the energy distribution \(N(E, t) = 4\pi V^2 (dV/dE)F(V, t)\). The corresponding systematic acceleration rate is in the conventional form

\[
\frac{dE}{dt}_d = \frac{1}{V^2} \frac{d}{dV} V^2 D_V \frac{dE}{dV} = \frac{1}{V^2} \frac{d}{dV} V^3 D_V,
\]

(2)

and the fluctuation acceleration coefficient

\[
D_E = (\frac{dE}{dV})^2 D_V = V^2 D_V.
\]

(3)

We consider, however, a more general model comprising also a regular acceleration, particle leakage from the acceleration region, and the charge-changing processes. Correspondingly, the distribution function \(N_i(E)\) for the ions of charge \(Q = i - 1\) adheres to the following equation:

\[
\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial E} \left( \frac{dE}{dt}_i N_i \right) - \frac{\partial^2}{\partial E^2} (D_E N_i) + \frac{N_i}{\tau_{esc}} + n([S_i + z_i N_i - S_{i-1} N_{i-1} - \alpha_{i+1} N_{i+1}])
\]

\[= X_i \delta(t)(E - E_d),\]

(4)

where \(n\) is the ambient electron number density, \(S_i(T, E)\) the ionization rate coefficient for the transition from the ionization state \(i\) to the state \(i + 1\), and \(z_i(T, E)\) the recombination rate coefficient from the ionization state \(i\) to \(i - 1\) (\(S_i\) and \(z_i\) are in units of cm\(^3\) s\(^{-1}\)). The advection coefficient in the energy space is of the form

\[
\frac{dE}{dt} = \left( \frac{dE}{dt}_d \right) + \left( \frac{dE}{dt}_sh \right) - \left( \frac{dE}{dt}_c \right).
\]

(5)

\(^3\) The high-energy limit for the experimentally deduced charge states of solar Fe ions presently is 50–60 MeV nucleon\(^{-1}\) (Mazur et al. 1999; Cohen et al. 1999).

incorporating a systematic energy gain produced by stochastic acceleration (eq. [2]), a regular acceleration rate, and the Coulomb energy losses, respectively. Recall that the absolute value of Coulomb losses in plasma, \((dE/dt)_c\), reaches a maximum value, \((dE/dt)_c\text{ max}\), at the energy \(E_{max}\) depending on the ambient electron temperature (e.g., Akhiezer et al. 1975, § 13.2). At coronal temperatures this energy falls into the SEP range:

\[
E_{max} = 0.355 \frac{T}{10^6 \text{ K}} \text{ MeV nucleon}^{-1},
\]

(6)

\[
\left( \frac{dE}{dt} \right)_c\text{ max} = 8.8 \times 10^{-4} \frac{Q^2}{A} \frac{n}{10^6 \text{ cm}^{-3}} \times \left( \frac{T}{10^6 \text{ K}} \right)^{-1/2} \text{ MeV nucleon}^{-1} \text{ s}^{-1},
\]

(7)

where \(T\) is the ambient electron temperature and \(Q\) and \(A\) are the energetic ion charge and mass numbers, respectively. We employ injection of the thermal charge distribution of Fe ions, \(X_i(T = 1.26 \times 10^6 \text{ K})\), at the injection energy \(E_0 = 20\) keV nucleon\(^{-1}\). An absorbing boundary is placed at 10 keV nucleon\(^{-1}\).

The systematic energy gain due to stochastic acceleration, \((dE/dt)_d\), is parameterized with characteristic acceleration time \(\tau_a\) and the power-law index \(S\):

\[
\left( \frac{dE}{dt} \right)_d = E_{\tau a} \left( \frac{E}{E_{\tau a}} \right)^S.
\]

(8)

at \(E_a = 1\) MeV nucleon\(^{-1}\). In the case of particle scattering by Alfvén waves with the turbulence spectral index \(q\), the particle diffusion coefficient in the momentum space \(D_p \propto p^{2-q}\), and the stochastic acceleration rate \((dE/dt)_d \propto V^{q-1}\) (eq. [2]). On the other hand, the regular shock-acceleration time \(\tau_{sh} \propto D_{||} \propto p^{3-q}\) (e.g., Toptygin 1985, § 18). Correspondingly, the \(V\)-dependence of the regular acceleration

\[
\text{Fig. 1.—Equilibrium energy curves, } dE/dt = 0, \text{ depicting the Fe ion charge marked by the equality of the Coulomb energy losses and the energy gain due to acceleration } (S = \frac{1}{2})\text{, for different values of the parameter } n \times \tau \text{ shown next to the curves in units of } 10^{19} \text{ cm}^{-3} \text{ s}^{-1}. \text{ The equilibrium charge curve } (Q_{eq}) \text{ represents the mean charge of Fe ions when ionization cancels recombination at a fixed energy of ions. Relative position of the two curves, } dE/dt = 0 \text{ and } Q_{eq}, \text{ dictates the shape of the } Q-E \text{ region filled with particles.}
\]
rate, \((dE/dt)_\text{sh} = E/\tau_1 \propto V^{-1}\), coincides with that for stochastic acceleration. For this reason, and also for simplicity’s sake, we employ similar parameterization for regular acceleration and for stochastic acceleration. Thus, we adopt the regular acceleration rate, \((dE/dt)_\text{sh}\), in the form

\[
\left(\frac{dE}{dt}\right)_\text{sh} = \frac{E_1}{\tau_1} \left(\frac{E}{E_1}\right)^s,
\]

where \(\tau_1\) is the characteristic time of regular acceleration. Then one can introduce a combined acceleration time, \(\tau\), defined by the formula

\[
\tau^{-1} = \tau_1^{-1} + \tau_1^{-1},
\]

corresponding to the sum of the stochastic and regular acceleration rates. We intend to study how acceleration mechanisms of different degrees of stochasticity affect the ion charge states observed in the interplanetary medium. With this in mind, we introduce a parameter \(a\) governing contribution of stochastic acceleration to the total acceleration rate:

\[
a = \frac{\tau}{\tau_1}.
\]

Using equations (2) and (3) and then equations (8) and (11), one can find also parameterization for the fluctuation acceleration coefficient:

\[
D_E = \frac{1}{V} \int \left(\frac{dE}{dt}\right)_\text{sh} V^2 dV = \frac{2a}{2S + 3} \frac{E_1^2}{\tau} \left(\frac{E}{E_1}\right)^{S+1}.
\]

We adopt the parameter value \(S = \frac{3}{2}\) corresponding to the Kraichnan phenomenology\(^4\) (except for Fig. 7, where \(S = 0\)).

We employ the time-integrated distribution of Fe ions inside the acceleration region,

\[
\Phi(E) = \int N(E, t) dt,
\]

and the time-integrated distribution in the interplanetary medium, \(\Phi(E)/\tau_\text{esc}\). Integrating equation (4) over the time yields at \(E > E_0\) the following equation for \(\Phi(E)\):

\[
\frac{d}{de} (e^s \Phi_i) - \frac{2a}{2S + 3} \left(\frac{e^{S+1} \Phi_i}{2S + 3}\right) + \frac{\tau}{\tau_\text{esc}} \Phi_i = (n \times \tau) \left[\alpha_{i+1} \Phi_{i+1} - (S_i + \alpha_i) \Phi_i + S_{i-1} \Phi_{i-1} + \frac{d}{de} \delta_e \Phi_i\right],
\]

\(^4\) The Kraichnan spectrum, \(q = 3/2\), was previously employed in a number of SEP-oriented studies (e.g., Miller & Roberts 1995).
Fig. 3.—Model iron distribution functions for different values of the parameter \( n \times \tau \) (columns) and the acceleration-type parameter \( a \) (rows). Contribution of stochastic acceleration rises from \( a = 0 \) in rows 1 and 2, via \( a = 0.2 \) in row 3, to \( a = 0.95 \) in row 4. The first row shows distribution of ions inside the acceleration region at \( \tau_{acc} \to \infty \); the rest of the panels show distribution of escaping ions at \( \tau_{esc} \), according to eq. (15). Parameter \( n \times \tau = (1, 2, 3) \times 10^{10} \) cm\(^{-3}\) s in columns A, B, and C, respectively. The colors indicate the value of \( \log \Phi(E) \) (first row) or \( \log \Phi(E)/\tau_{esc} \) (rows 2–4). The colors range logarithmically from red at maximum value down to violet. The step between the neighboring colors is \( \log \Phi = 1 \) in frames 1A and 1B, and 1.5 in frames 1C and 2A–4C. The equilibrium charge is shown as a reference (the same black curve in all frames).

where we have introduced the dimensionless energy \( \varepsilon = E/E_1 \) and \( \dot{\varepsilon}_c E_1 = (dE/dt)_c/n \), that is, the absolute value of the Coulomb energy losses per one electron of the ambient plasma.

Our study is aimed at revealing properties of the system controlled by the density proportional terms in the right-hand side of equation (14), when acceleration is not very fast as compared to Coulomb losses. We also intend to investigate the role of stochasticity/regularity of the acceleration mechanism in forming the charge-state distributions. For the goals of our study, we unify and simplify the description of acceleration. We suggest that the acceleration model parameters are independent of the ion charge, whereas the rates of the charge-changing processes and Coulomb energy losses are kept charge dependent. For instance, we do not account for a weak dependence of the stochastic acceler-
ation rate on the ion charge \([(dE/dt)_a \propto Q^{1/2} at q = 3/2; e.g., Möbius et al. 1982], and similarly for regular acceleration. Instead, we choose equal spectral indexes for ions accelerated in all cases, except a change in the spectrum due to the Coulomb losses effect. Such a choice allows one to observe the net effect of stochasticity on charge-state distributions, when the parameter \(a\) rises. Thus, we have to fix the ratio \(\tau/\tau_{\text{esc}}\), because this ratio dominates the spectral index of escaping particles in the absence of Coulomb losses. Here we suggest a power-law spectrum of the escaping Fe ions,\(^5\) \(\Phi(E)/\tau_{\text{esc}} \propto E^{-3/2}\). At \(S = \frac{1}{2}\), this implies the escape-to-acceleration time ratio in the form

\[
\frac{\tau_{\text{esc}}}{\tau} = \frac{14}{7 - 2a} e^{3/4}.
\] (15)

We are about to study the charge-changing acceleration model with two variable parameters: (1) the contribution of stochastic acceleration to the total acceleration rate, parameter \(a\) (eq. [11]), and (2) the density \(n\) x acceleration-time product, \(n \times \tau\), that appears as a common coefficient in front of the terms describing the charge-changing processes and the Coulomb energy losses (eq. [14]). Those terms act in concert and cannot be decoupled. Performance of the charge-changing processes has been formulated by Kocharov et al. (2000), but the proton-impact ionization cross section is updated (Barghouty 2000; see also our Appendix). We consider completely ionized hydrogen plasma at log \(T = 6.1\).

3. RESULTS OF MONTE CARLO SIMULATIONS

Solutions of equation (14) are found with a Monte Carlo simulation technique. We keep track of Fe ions and regularly register them during their trip in the charge-energy plane. The resultant distribution function, \(\Phi(E)\), is proportional to the time spent by an average Fe ion in a particular area of the plane.

It is helpful to start with a description of the system in terms of the phase trajectories. In the case of fast regular acceleration, ions typically travel along simple rising trajectories in the \(Q\)-\(E\) plane (phase plane of the system). In an attempt to reach the equilibrium charge state, one may slow down the acceleration, and the Coulomb energy losses become important in a significant portion of the \(Q\)-\(E\) plane. One can deduce the equilibrium energy curve in which the systematic energy gain due to acceleration, \((E_1/E_2)\(\times\(E/E_2)\)\), is exactly canceled by the Coulomb deceleration, so that \(dE/dt = 0\). The equilibrium energy curves are shown in Figure 1 for different values of the \(n \times \tau\) product (numbers next to the curves are in units of \(10^{10} \text{ cm}^{-3} \text{ s}\)). Coulomb losses are dominant in the areas above the equilibrium energy curves. It turns out that topology of the typical phase trajectories depends on the positioning of the \(dE/dt = 0\) curve in respect to the equilibrium charge curve,\(^6\) \(Q_{\text{eq}}(E)\). If the equilibrium energy curve is situated far above the equilibrium charge curve, ions are accelerated along the simple rising trajectories, but the situation changes when curves approach each other.

In a preliminary set of simulations, we neglected the leaky box term \(\Phi_0\) in equation (14) and considered phase trajectories and the distribution function \(\Phi(E)\) inside the acceleration region. Corresponding results are shown in Figure 2 and in the first row of Figure 3. The broken line in Figure 2 exemplifies a phase trajectory of Fe during regular (\(a = 0\)) acceleration, when the equilibrium energy curve touches the equilibrium charge curve. If the value of \(n \times \tau\) were less than \(1 \times 10^{10} \text{ cm}^{-3} \text{ s}\), ions would be accelerated only along simple trajectories like 0–1–3–4. However, at the value \(n \times \tau = 2.5 \times 10^{10} \text{ cm}^{-3} \text{ s}\) shown in Figure 2, a particle gets temporally trapped in region 2 and then may either continue acceleration along the rising route 3–4 or after ionization enter the deceleration region, \(dE/dt < 0\), and continue along 5–6 back to region 2, and so on. The color scale depicts the time that an average particle spends in different areas of the phase plane (more specifically log \(\Phi_0\)). The first row of Figure 3 illustrates how the ion distribution evolves as the acceleration weakens (the \(n \times \tau\) parameter increases). The simple rising trajectories dominate at \(n \times \tau = 1 \times 10^{10} \text{ cm}^{-3} \text{ s}\) (Fig. 3, 1A). They are supplemented with a deceleration movement at \(n \times \tau = 2 \times 10^{10} \text{ cm}^{-3} \text{ s}\) (Fig. 3, 1B). High-charge ions appear at low energies. Finally, at \(n \times \tau = 3 \times 10^{10} \text{ cm}^{-3} \text{ s}\), particles

\(^5\) The value of the ion spectrum power-law index is not important for our study. We choose the value of 3/2, which is not far from the mean value observed in the 0.03–100 MeV nucleon\(^{-1}\) range during the 1997 November 6 event (Mason et al. 1999; Torsti et al. 2000).

\(^6\) The mean equilibrium charge, \(Q_{\text{eq}}(E)\), is the ionic charge averaged over the equilibrium distribution, \(N_{\text{eq}}(E)\), that is defined as a solution of the balance equation at a fixed value of energy, \((S_+ + a)N_{\text{eq}}(E) = S_+ N_{\text{esc}}(E) + a_{+1} N_{\text{esc}}(E)\). This equation might come from eq. (4) if all other terms in that equation were negligibly small, including in particular the Coulomb losses term (also eq. [14]).
are not able to penetrate the Coulomb losses barrier and are kept at sub-MeV energies (Fig. 3, 1C). The rest of Figure 3 (rows 2–4) shows the distribution of escaping particles, $\Phi_{\gamma}/\tau_{\text{esc}}$, with the escape time adopted according to equation (15) at different values of the parameter $n \times \tau$ (columns) and different contributions of stochastic acceleration, parameter $a$ of equation (11) (rows). The particle escape from the acceleration region brings about the formation of slope toward the high energies (e.g., frame 2A vs. 1A). Stochastic acceleration blurs the low-energy edge of the distribution (frame 3A vs. 2A) and causes the particles to penetrate through the Coulomb losses barrier (3C vs. 2C).

The mean nonequilibrium charge states of Fe ions, $\langle Q \rangle$, are shown in Figure 4 for different values of the acceleration model parameters $n \times \tau$ and $a$ (member curves “a” through “c”). Curves “d” in Figure 4 additionally illustrate the mean equilibrium charge $Q_{\text{eq}}(E)$ (heavy solid curve) and an “erroneous” calculation of $\langle Q \rangle$, when the Coulomb energy losses were artificially turned off (dashed curve). It is seen that the nonequilibrium mean charge states are not very sensitive to the relative contribution of stochastic acceleration. A difference between stochastic acceleration and regular acceleration is more pronounced at larger values of $n \times \tau$, when $\langle Q \rangle$ tends to approach $Q_{\text{eq}}(E)$. However, $\langle Q \rangle$ could not reproduce $Q_{\text{eq}}(E)$ without a strong reduction in the number of accelerated particles caused by the Coulomb energy losses at sub-MeV energies (Fig. 5). Dashed curve “d” in Figure 4 might be attained without Coulomb energy losses included, if the effective acceleration were possible at a very high value of the parameter $n \times \tau = 4 \times 10^{11}$ cm$^{-3}$ s ($a = 0.95$). In such a case, the mean charge $\langle Q \rangle$ would come close to $Q_{\text{eq}}(E)$, although the steps in $Q_{\text{eq}}(E)$ still would be smoothed. However, with the real Coulomb losses included, there are very few ions accelerated beyond $\sim 1$ MeV nucleon$^{-1}$ if $n \times \tau \geq 10^{11}$ cm$^{-3}$ s.$^2$

Figure 4 indicates that the contribution of stochastic acceleration (parameter $a$) has a little effect on the mean charge states, but shapes of the charge-state distributions are different (Fig. 3, three lower rows). Figure 6 exemplifies the corresponding charge-state distributions in the 0.25–0.4 MeV nucleon$^{-1}$ energy channel for different values of parameter $a$. A high-$Q$ tail is clearly seen if the stochastic component of acceleration is strong ($a = 0.95$).

In the final set of simulations, we demonstrate how relative weakness of recombination in plasma affects Fe charge states during slowing down. For this purpose, we have performed a numerical experiment, shown in Figure 7. We consider a regular acceleration ($a = 0$) up to a maximum energy, $E_f$, followed by a pure Coulomb slowing down, when the acceleration has been abruptly turned off. Corresponding phase trajectories for the mean charge $\langle Q \rangle$ are shown with solid lines at several different values of $E_f$. It turns out that the phase trajectories represent a kind of hysteresis, because the Coulomb energy losses in plasma are fast as compared to the recombination and ions slow down, keeping a high charge state that seems appropriate to a higher energy (the trajectories are not closed, because we do not consider very low energies, where the Fe ions return to the thermal charge states). In an additional simulation, we artificially reduced the Coulomb losses by a factor of 10 and then by a factor of 40 (Fig. 7, dashed and dotted curves, respectively). It is seen that if the energy losses were $\geq 40$ fold weaker (as compared to recombination), the ions would run through a series of nearly equilibrium charge states, similar to the energetic ion behavior in neutral media. However, it does not happen with Fe ions in the coronal plasma.

4. DISCUSSION AND CONCLUSIONS

We have considered a unified model of ion acceleration incorporating both regular and stochastic acceleration along with the density-dependent processes, Coulomb deceleration, ionization, and recombination (eq. [14]). We find that acceleration type in itself, either regular or stochastic, has a little effect on the nonequilibrium mean charge of iron as a function of energy (Fig. 4), if the acceleration models are uniform in all other parameters. However, a type of acceleration may leave its imprints in the shape of the ion charge distribution. Stochastic acceleration typically produces an extended high-charge wing in the distribution, whereas the regular acceleration distributions are more symmetric and narrow (Fig. 6). However a high-charge wing may be also produced by regular acceleration, if its rate is comparable with the rate of Coulomb losses (Fig. 3, 2B). Note that neglecting charge dependence in the acceleration rate ($\gamma$ 2) slightly deforms ion phase trajectories. If the charge dependence were included, the nonequilibrium mean charge curves in Figure 4 would rise slightly less steeply, farther from the equilibrium-charge curve $Q_{\text{eq}}(E)$.

There were a number of investigations of particle acceleration competing with strong Coulomb losses. In particular, Coulomb losses played a significant role in the acceleration models attempting to explain the energy dependence of the
\[ n \times \tau_C = 4 \times 10^{10} \frac{A}{Q^2} \left( \frac{T}{10^6 \text{ K}} \right)^{3/2} \text{ cm}^{-3} \text{ s}, \]

where \( \tau_C = E_{\text{max}}/(dE/dt)_{\text{c max}} \) is the characteristic deceleration time at the energy \( E_{\text{max}} \) (see eqs. [6] and [7]). A typical estimate for the Fe ions is \( n \times \tau_C \approx (1-2) \times 10^{10} \text{ cm}^{-3} \text{ s} \). This is close to the time it takes for an ion moving with a constant speed to get stripped up to the equilibrium charge \( Q_{\text{eq}} \) (Fig. 6 of Kocharov et al. 2000).

In the case of ions stopping in neutral gases, an equilibrium charge appropriate to the ion velocity was introduced, being a universal function of the velocity and the nucleonic charge (see Reames et al. 1999). On the other hand, long ago Korchak (1980) raised the question as to whether the equilibrium charge can be attained in the coronal plasma, when acceleration competes with Coulomb losses. The mean charge versus energy curve should approach the equilibrium charge \( Q_{\text{eq}}(E) \) as the density \( n \times \tau_{\text{acc}} \) product is increased (e.g., Ostryakov et al. 2000b). 7 We have carefully studied the case of slow acceleration, where the acceleration rate is compa-
PROTON-IMPACT IONIZATION CROSS SECTION

Recently, Barghouty (2000) described a simple procedure to estimate proton-impact ionization cross sections over the energy range up to tens of MeV nucleon\(^{-1}\). The cross section is taken to be the sum over partial cross sections corresponding to different electron subshells \(j\). The procedure connects, in the first Born approximation, the partial-proton-impact cross section \(\sigma_p\) to the known electron-impact cross section \(\sigma_e\). After straightforward algebra, we cast the partial-proton-impact cross section into the form

\[
\sigma_p(u, I_j) = \frac{(1 + 4u)^3}{4u(16u^2 + 4u - 2)} \sigma_e(u, I_j),
\]

where

\[
u = \frac{m_e E}{M_p I_j},
\]

and

\[
\sigma_p(u, I_j) \approx Q_{eq}(E).
\]

Coulomb energy losses in the solar corona may be important also in context of \(\gamma\)-ray observations (e.g., Emslie, Brown, & MacKinnon 1997 and references therein). The equilibrium mean charge \(Q_{eq}(E)\) is usually employed for estimates of ion stopping in calculations of secondary emissions generated by energetic ions in astrophysical plasmas (e.g., Tatischeff, Ramaty, & Kozlovsky 1998). The pattern of ion behavior shown in Figure 7 might be instructive for such an application. In Figure 7, ions have been accelerated up to the energy \(E_f\), leave the acceleration with a nonequilibrium charge \(\langle Q \rangle < Q_{eq}(E_f)\), and then experience concurrent charge change and Coulomb energy losses in plasma. After the fast stripping in the beginning of the stopping phase, the ion charge approaches \(\langle Q \rangle \approx Q_{eq}(E_f/2)\) and then changes very gradually. Hence, the equilibrium charge at half the initial energy for stopping would provide a reasonable estimate for the stopping power, and this effective charge seems independent of the current energy, \(E\), until the ion has lost most of its initial energy.

The most dramatic finding of the present study is the possibility of dynamic cycles in the charge-energy plane. During continual acceleration, the dynamical \(Q-E\) cycles become possible (Fig. 2), when the minimum of the equilibrium energy curve, \(dE/dt = 0\), appears in the Fe ion charge range (Fig. 1). The latter happens when the value of the density \(\times\) time product is in the range \(10^{10}-10^{11} cm^{-3}\) s, the possible range for ions accelerated in the low and middle corona of the Sun (e.g., Kocharov & Kovaltsov 1986; Kocharov et al. 2000; Ostryakov et al. 2000b). The \(Q-E\) loops may appear not only in the particular particle trajectories (Fig. 2) but also in the mean charge trajectories (Fig. 7), when strong Coulomb losses drag stripped ions back to low energies. The absence of the universal \(Q(E)\) curve makes resulting charge-energy distributions of accelerated ions sensitive to the scenario of their acceleration and transport. For this reason, one can expect in future a wide variety of charge-energy profiles observed in solar energetic particles and calculated in more sophisticated theoretical models.

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$j$ is the index of the subshell with the ionization potential $I_j$, $m_e/M_p$ is the electron-to-proton mass ratio, and $u_e$ is related to $u$ as

$$u_e = \left(1 + \frac{1}{4u}\right)^2 u. \quad (A3)$$

The electron-impact cross section $\sigma_{ej}$ was given by Arnaud & Raymond (1992).

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